Studiare la funzione e disegnare il grafico :

$$f(x) = x - \frac{e^{2x}}{e^{2x}+1}$$

1. Dominio : $\forall x \in \Re : x \in \Re$

2. Intersezione Assi :

$$\begin{cases} y = x - \frac{e^{2x}}{e^{2x}+1} \\ x = 0 \end{cases} \Rightarrow \begin{cases} y = -\frac{1}{2} \\ x = 0 \end{cases}$$

3. Limiti :

$$\lim_{x \to -\infty^-} x - \frac{e^{2x}}{e^{2x}+1} = -\infty$$

$$\lim_{x \to +\infty^-} x - \frac{e^{2x}}{e^{2x}+1} = \lim_{x \to +\infty^-} x - \frac{e^{2x}}{e^{2x}\left(1+\frac{1}{e^{2x}}\right)} = \lim_{x \to +\infty^-} x - \frac{1}{1+\frac{1}{e^{2x}}} = +\infty$$

4. Asintoti :

$$Per \quad x \to -\infty \qquad m = \lim_{x \to -\infty} \frac{x - \frac{e^{2x}}{e^{2x}+1}}{x} = \lim_{x \to -\infty} 1 - \frac{e^{2x}}{x\left(e^{2x}+1\right)} = 1$$

$$q = \lim_{x \to -\infty} x - \frac{e^{2x}}{e^{2x}+1} - x = \lim_{x \to -\infty} -\frac{e^{2x}}{e^{2x}+1} = 0$$

$$Per \quad x \to +\infty \qquad m = \lim_{x\to+\infty} \frac{x - \dfrac{e^{2x}}{e^{2x}+1}}{x} = \lim_{x\to+\infty} 1 - \frac{e^{2x}}{xe^{2x}\left(1+\dfrac{1}{e^{2x}}\right)} = \lim_{x\to+\infty} 1 - \frac{1}{x\left(1+\dfrac{1}{e^{2x}}\right)} = 1$$

$$q = \lim_{x\to+\infty} x - \frac{e^{2x}}{e^{2x}+1} - x = \lim_{x\to+\infty} -\frac{e^{2x}}{e^{2x}+1} = \lim_{x\to+\infty} -\frac{e^{2x}}{e^{2x}\left(1+\dfrac{1}{e^{2x}}\right)} = \lim_{x\to+\infty} -\frac{1}{1+\dfrac{1}{e^{2x}}} = -1$$

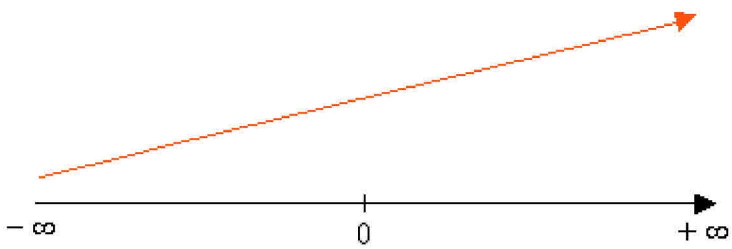
quindi le equazioni degli asintoti sono:

$$\boxed{y = x \qquad , \qquad y = x - 1}$$

5. Derivata 1^ :

$$f'(x) = 1 - \frac{2e^{2x}\left(e^{2x}+1\right) - 2e^{4x}}{\left(e^{2x}+1\right)^2} = 1 - \frac{2e^{2x}}{\left(e^{2x}+1\right)^2} = \frac{\left(e^{2x}+1\right)^2 - 2e^{2x}}{\left(e^{2x}+1\right)^2} = \frac{2e^{4x}+1}{\left(e^{2x}+1\right)^2}$$

$$f'(x) > 0 \qquad \Rightarrow \qquad \forall x \in \Re$$

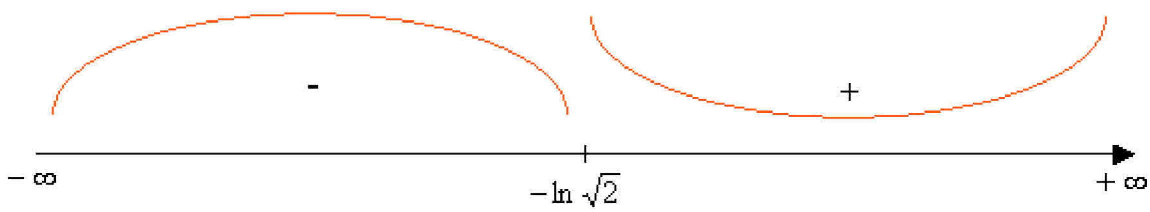


6. Derivata 2^ :

$$f''(x) = \frac{8e^{4x}\left(e^{2x}+1\right)^2 - 4e^{2x}\left(e^{2x}+1\right)\left(2e^{4x}+1\right)}{\left(e^{2x}+1\right)^4} = \frac{4e^{2x}\left(e^{2x}+1\right)\left[2e^{2x}\left(e^{2x}+1\right) - 2e^{4x} - 1\right]}{\left(e^{2x}+1\right)^4} =$$

$$= \frac{4e^{2x}\left[2e^{2x}-1\right]}{\left(e^{2x}+1\right)^3}$$

$$f''(x) > 0 \quad \Rightarrow \quad 2e^{2x} - 1 > 0 \quad \Rightarrow \quad e^{2x} > \frac{1}{2} \quad \Rightarrow \quad x > \frac{1}{2}\ln\frac{1}{2} \quad \Rightarrow \quad x > -\ln\sqrt{2}$$



L'ordinata del flesso : $f\left(-\ln\sqrt{2}\right) = -\ln\sqrt{2} - \frac{e^{-2\ln\sqrt{2}}}{e^{-2\ln\sqrt{2}}+1} = -\ln\sqrt{2} - \frac{1}{3}$

Il grafico :