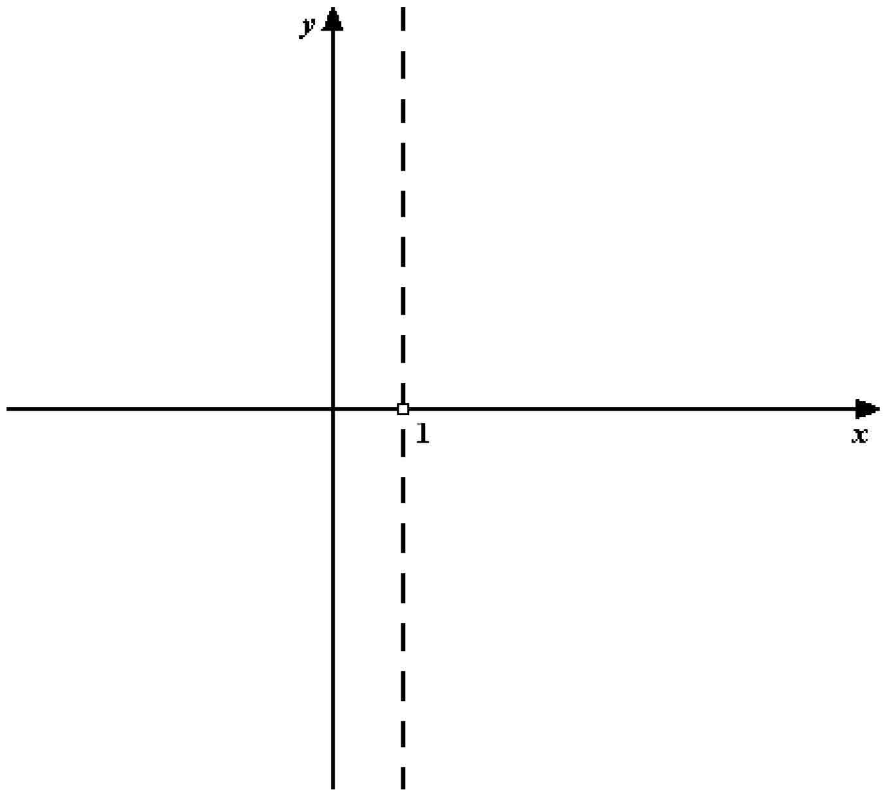
Studiare e rappresentare graficamente la funzione

$$f(x) = \frac{x^2 - 3x}{|x-1|}$$

Svolgimento :

1. $C.E. \Rightarrow \forall x \in \Re : x \neq 1$



2. Intersezioni Assi

$$\begin{cases} y = \dfrac{x^2-3x}{|x-1|} \\ x = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases} \Rightarrow A(\,0\,,0\,)$$ punto d'intersezione con l'asse y .

$$\begin{cases} \dfrac{x^2-3x}{|x-1|} = 0 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x^2-3x = 0 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} , \begin{cases} x = 3 \\ y = 0 \end{cases} \Rightarrow B(\,3\,,0\,)$$

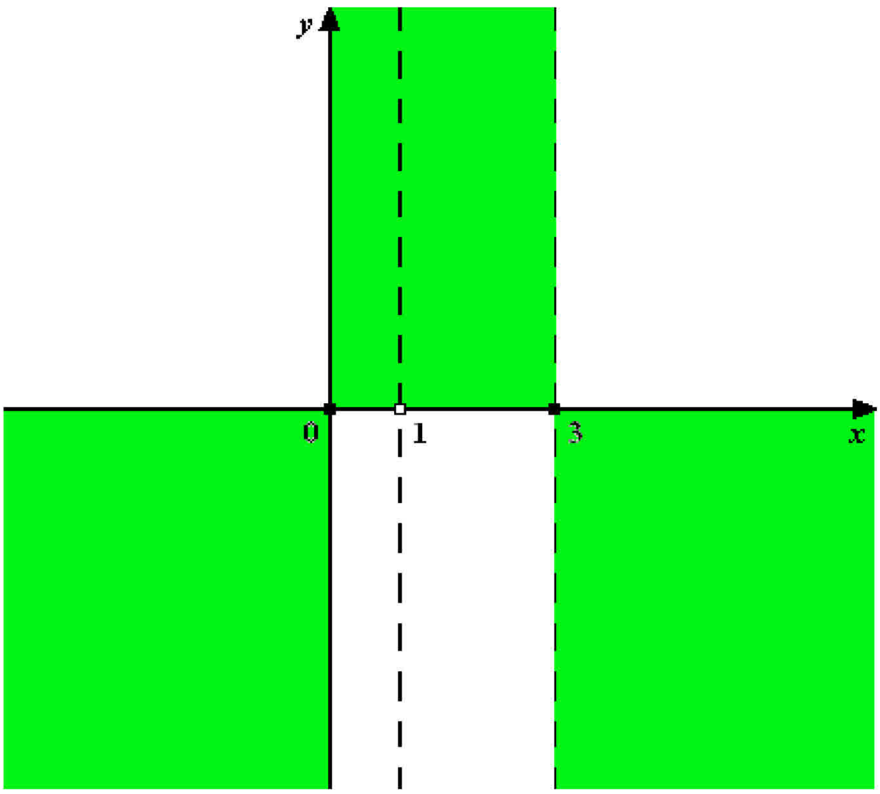
punto d'intersezione con l'asse x .

3. Segno della Funzione : $f(x) > 0$

poiché il valore assoluto esprime una quantità positiva

si ha : $x^2 - 3x > 0 \Rightarrow x < 0 \;,\; x > 3$

N.B. Le regioni piane contrassegnate dal colore verde escludono la presenza della funzione , avendone determinato sopra il segno.



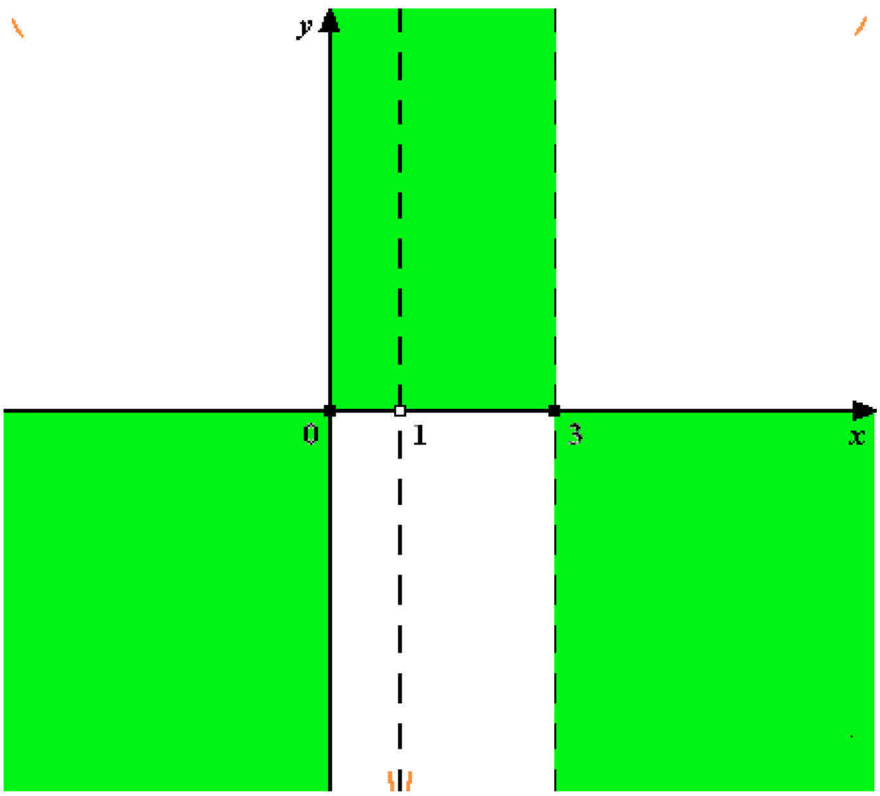
4. Limiti

$$\lim_{x \to 1} \frac{x^2 - 3x}{|x-1|} = -\infty$$

$$\lim_{x \to -\infty} \frac{x^2 - 3x}{|x-1|} = \left(\frac{+\infty}{+\infty}\right) \Rightarrow \lim_{x \to -\infty} \frac{x^2 - 3x}{1-x} = \lim_{x \to -\infty} \frac{x^2\left(1 - \frac{3}{x}\right)}{x^2\left(\frac{1}{x^2} - \frac{1}{x}\right)} = +\infty$$

$$\lim_{x \to +\infty} \frac{x^2 - 3x}{|x-1|} = \left(\frac{+\infty - \infty}{+\infty}\right) \Rightarrow \lim_{x \to +\infty} \frac{x^2 - 3x}{x-1} = \lim_{x \to +\infty} \frac{x^2\left(1 - \frac{3}{x}\right)}{x^2\left(\frac{1}{x} - \frac{1}{x^2}\right)} = +\infty$$

La rappresentazione grafica dello studio dei limiti :



5. Asintoti

$\boxed{x = 1}$ **asintoto verticale**

verifica esistenza asintoti obliqui : $y = mx + q$

$$m = \lim_{x \to -\infty} \frac{f(x)}{x} = \left(\frac{+\infty}{-\infty} \right) \Rightarrow \lim_{x \to -\infty} \frac{x^2 - 3x}{x - x^2} = \lim_{x \to -\infty} \frac{x^2\left(1 - \frac{3}{x}\right)}{x^2\left(\frac{1}{x} - 1\right)} = -1$$

$$q = \lim_{x \to -\infty} f(x) - mx = \lim_{x \to -\infty} \frac{x^2 - 3x}{1 - x} + x = (+\infty - \infty) \Rightarrow \lim_{x \to -\infty} \frac{-2x}{1 - x} = \left(\frac{+\infty}{+\infty} \right)$$

$$q = \lim_{x \to -\infty} \frac{-2x}{x\left(\frac{1}{x} - 1\right)} = 2$$

quindi per $x \to -\infty$ $\Rightarrow$ $y = -x + 2$ **asintoto obliquo** **,**

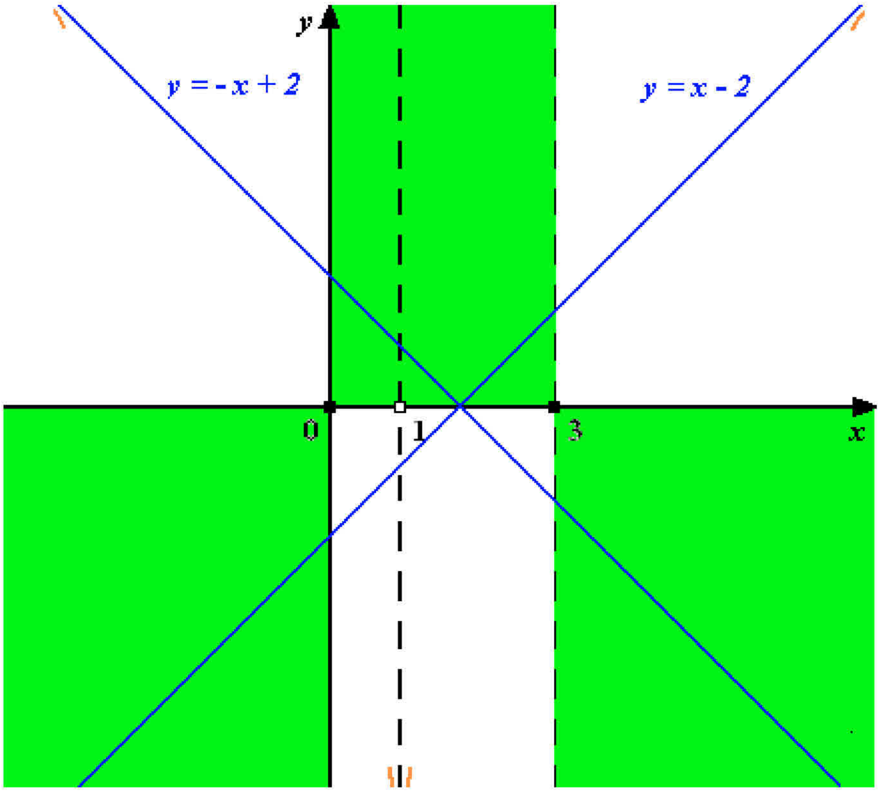
$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \left(\frac{+\infty}{+\infty} \right) \Rightarrow \lim_{x \to +\infty} \frac{x^2 - 3x}{x^2 - x} = \lim_{x \to +\infty} \frac{x^2\left(1 - \frac{3}{x}\right)}{x^2\left(1 - \frac{1}{x}\right)} = 1$$

$$q = \lim_{x \to +\infty} f(x) - mx = \lim_{x \to +\infty} \frac{x^2 - 3x}{x - 1} - x = (+\infty - \infty) \Rightarrow \lim_{x \to +\infty} \frac{-2x}{x - 1} = \left(\frac{-\infty}{+\infty} \right)$$

$$q = \lim_{x \to +\infty} \frac{-2x}{x\left(1 - \frac{1}{x}\right)} = -2$$

quindi per $x \to +\infty$ $\Rightarrow$ $y = x - 2$ **asintoto obliquo**

La rappresentazione grafica dello studio degli asintoti :

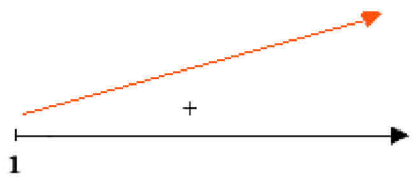


6. Derivata 1^

Per $x-1>0 \quad \rightarrow \quad x>1 \quad \Rightarrow \quad f(x)=\dfrac{x^2-3x}{x-1}$

$$f'(x) \quad = \quad \frac{(2x-3)(x-1)-(x^2-3x)}{(x-1)^2} \quad = \quad \frac{x^2-2x+3}{(x-1)^2}$$

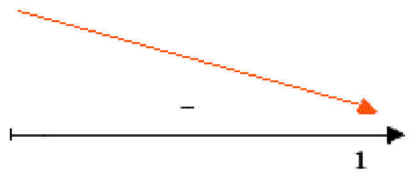
$$f'(x) \quad > \quad 0 \quad \Rightarrow \quad x^2-2x+3>0 \quad \Rightarrow \quad \forall x \in \Re : x>1$$



Per $x-1<0 \quad \rightarrow \quad x<1 \quad \Rightarrow \quad f(x)=\dfrac{x^2-3x}{1-x}$

$$f'(x) = \frac{(2x-3)(1-x)+(x^2-3x)}{(1-x)^2} = \frac{-x^2+2x-3}{(1-x)^2}$$

$$f'(x) > 0 \quad \Rightarrow \quad x^2-2x+3<0 \quad \Rightarrow \quad \forall x \in \Re$$

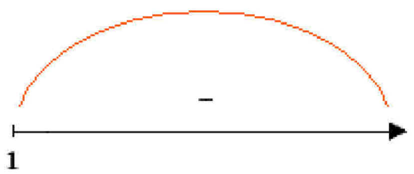


7. Derivata 2^

Per $x-1>0 \quad \rightarrow \quad x>1 \quad \Rightarrow \quad f'(x)=\dfrac{x^2-2x+3}{(x-1)^2}$

$$f''(x) = \frac{(2x-2)(x-1)^2-2(x-1)(x^2-2x+3)}{(x-1)^4} = \frac{2(x-1)(x^2-2x+1-x^2+2x-3)}{(x-1)^4} = \frac{-4}{(x-1)^3}$$

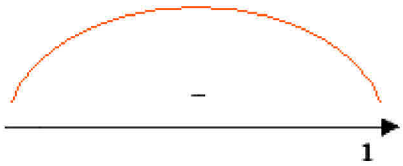
$$f''(x) > 0 \quad \Rightarrow \quad \forall x \in \Re : x<1$$



Per $\quad x-1<0 \qquad \rightarrow \qquad x<1 \qquad \Rightarrow \qquad f'(x)=\frac{-x^2+2x-3}{(1-x)^2}$

$$f''(x) \; = \; \frac{(-2x+2)(1-x)^2+2(1-x)(-x^2+2x-3)}{(1-x)^4} \; = \; \frac{-4}{(1-x)^3}$$

$f''(x) \; > \; 0 \qquad \Rightarrow \quad \forall x \in \Re$



Il grafico :