1. Data la funzione

$$f(x)=\frac{e^{3x+1}}{x^2-2x}$$

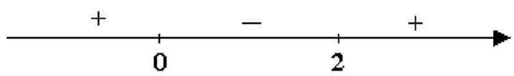
1. $C.E. \Rightarrow \forall\, x \in \Re : \quad x \neq 0 \quad , \quad x \neq 2$

2. Intersezioni Assi

$$\begin{cases} y=\dfrac{e^{3x+1}}{x^2-2x} \\ y=0 \end{cases} \Rightarrow \begin{cases} e^{3x+1}=0 \\ y=0 \end{cases} \Rightarrow \begin{cases} \not\forall x \in \Re \\ y=0 \end{cases}$$

3. Segno della Funzione

$f(x) > 0 \qquad x^2-2x > 0 \Rightarrow x<0 \; , \; x>2$



4. Limiti

$$\lim_{x\to 0^-}\frac{e^{3x+1}}{x^2-2x}=+\infty \quad , \quad \lim_{x\to 0^+}\frac{e^{3x+1}}{x^2-2x}=-\infty$$

$$\lim_{x\to 2^-}\frac{e^{3x+1}}{x^2-2x}=-\infty \quad , \quad \lim_{x\to 2^+}\frac{e^{3x+1}}{x^2-2x}=+\infty$$

$$\lim_{x\to -\infty}\frac{e^{3x+1}}{x^2-2x}=0$$

$$\lim_{x\to +\infty}\frac{e^{3x+1}}{x^2-2x}=\left(\frac{+\infty}{+\infty}\right) \Rightarrow H \lim_{x\to +\infty}\frac{3e^{3x+1}}{2x-2}=\left(\frac{+\infty}{+\infty}\right) \Rightarrow H \lim_{x\to +\infty}\frac{9e^{3x+1}}{2}=+\infty$$

5. Asintoti

$x = 0 \quad , \quad x = 2$	**asintoti verticale**	,	$y = 0$	**asintoto orizzontale**

Verifica dell'esistenza dell'asintoto obliquo , $y = mx + q$, per $x \to +\infty$

$$m = \lim_{x \to +\infty} \frac{e^{3x+1}}{x^2 - 2x} \cdot \frac{1}{x} = \left(\frac{+\infty}{+\infty} \right) \Rightarrow \lim_{x \to +\infty} \frac{e^{3x+1}}{x^3 - 2x^2} = \left(\frac{+\infty}{+\infty} \right)$$

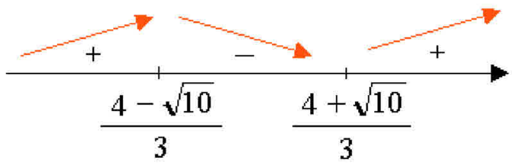
$$H \lim_{x \to +\infty} \frac{3e^{3x+1}}{3x^2 - 4x} = \left(\frac{+\infty}{+\infty} \right) \Rightarrow H \lim_{x \to +\infty} \frac{9e^{3x+1}}{6x - 4} = \left(\frac{+\infty}{+\infty} \right) \Rightarrow H \lim_{x \to +\infty} \frac{27e^{3x+1}}{6} = +\infty$$

non esiste quindi l'asintoto obliquo.

6. Derivata 1^

$$f'(x) = \frac{3e^{3x+1}(x^2 - 2x) - e^{3x+1}(2x - 2)}{(x^2 - 2x)^2} = \frac{e^{3x+1}(3x^2 - 6x - 2x + 2)}{(x^2 - 2x)^2} = \frac{e^{3x+1}(3x^2 - 8x + 2)}{(x^2 - 2x)^2}$$

$$f'(x) > 0 \quad \Rightarrow \quad 3x^2 - 8x + 2 > 0 \quad \Rightarrow \quad x < \frac{4 - \sqrt{10}}{3} \quad , \quad x > \frac{4 + \sqrt{10}}{3}$$



Riassumendo :

La $f(x)$ risulta **continua** in : $x \in \left]-\infty, 0\right[\cup \left]0, 2\right[\cup \left]2, +\infty\right[$; $f(x)$ risulta **derivabile** in : $x \in \left]-\infty, 0\right[\cup \left]0, 2\right[\cup \left]2, +\infty\right[$.

La $f(x)$ risulta **monotòna crescente** in : $x \in \left]-\infty, 0\right[\cup \left]0, \frac{4-\sqrt{10}}{3}\right[\cup \left]\frac{4+\sqrt{10}}{3}, +\infty\right[$; **monotòna decrescente** in : $x \in \left[\frac{4-\sqrt{10}}{3}, 2\right[\cup \left]2, \frac{4+\sqrt{10}}{3}\right]$.

La $f(x)$ assume **minimo relativo** in $x = \frac{4+\sqrt{10}}{3}$; **massimo relativo** in $x = \frac{4-\sqrt{10}}{3}$.

La $f(x)$ ha come **estremo superiore** $(\sup f = +\infty)$ **;** come **estremo inferiore** $(\inf f = -\infty)$.

Il grafico :