$$f(x)=1-\ln\frac{x^2-4}{x^2-9}$$

1. Dominio :

$$x<-3 \quad , \quad -2<x<+2 \quad , \quad x>+3$$

Poichè la funzione è **pari** , lo studio viene limitato al semipiano delle ascisse positive $(x \geq 0)$

2. Intersezione assi :

$$\begin{cases} y=1-\ln\frac{x^2-4}{x^2-9} \\ x=0 \end{cases} \Rightarrow \begin{cases} y=1-\ln\frac{4}{9} \\ x=0 \end{cases}$$

$$\begin{cases} y=1-\ln\frac{x^2-4}{x^2-9} \\ y=0 \end{cases} \Rightarrow \begin{cases} 1-\ln\frac{x^2-4}{x^2-9}=0 \\ y=0 \end{cases} \Rightarrow \begin{cases} \ln\frac{x^2-4}{x^2-9}=\ln e \\ y=0 \end{cases} \Rightarrow \begin{cases} (1-e)x^2=4-9e \\ y=0 \end{cases}$$

$$\begin{cases} x=+\sqrt{\frac{9e-4}{e-1}} \\ y=0 \end{cases}$$

3. Segno *f(x) > 0* :

$$1-\ln\frac{x^2-4}{x^2-9}>0 \Rightarrow \frac{x^2-4}{x^2-9}<e \Rightarrow \frac{(e-1)x^2+4-9e}{x^2-9}>0$$

$$\Rightarrow \quad 0<x<2 \quad , \quad x>+\sqrt{\frac{9e-4}{e-1}}$$

4. Limiti :

$$\lim_{x \to 2^-} \quad 1-\ln\frac{x^2-4}{x^2-9} \quad = \quad +\infty$$

$$\lim_{x \to 3^+} \quad 1-\ln\frac{x^2-4}{x^2-9} \quad = \quad -\infty$$

$$\lim_{x \to +\infty} \quad 1-\ln\frac{x^2-4}{x^2-9} \quad = \quad 1-\lim_{x \to +\infty} \ln\left[\frac{x^2\left(1-\frac{4}{x^2}\right)}{x^2\left(1-\frac{9}{x^2}\right)}\right] \quad = \quad 1$$

5. Asintoti :

$\boxed{x=2}$, $\boxed{x=3}$ **Asintoto verticale** $\boxed{y=1}$ **Asintoto orizzontale**

6. Derivata 1^ :

$$f'(x) \quad = \quad \frac{-\dfrac{2x\left(x^2-9\right)-2x\left(x^2-4\right)}{\left(x^2-9\right)^2}}{\dfrac{\left(x^2-4\right)}{\left(x^2-9\right)}} \quad = \quad \frac{10x}{\left(x^2-9\right)\left(x^2-4\right)}$$

$$f'(x) \quad > \quad 0 \qquad \forall\ x \in \Re : 0 < x < 2 \quad , \quad x > 3$$

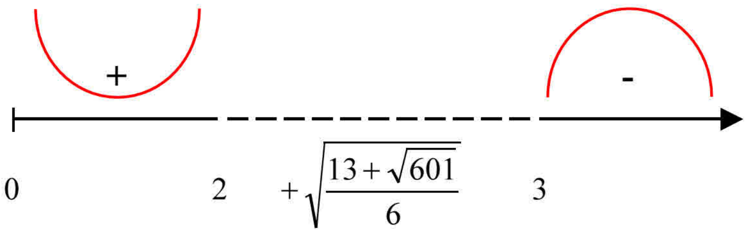
$$f(0) = 1 - \ln\frac{4}{9} \quad \Rightarrow \quad P\left(0 \,, 1 - \ln\frac{4}{9} \right) \quad \text{max}. \textit{relativo}$$

6. Derivata 2^ :

Allo stesso modo :

$$f''(x) \;=\; \frac{10(x^2-9)(x^2-4) - 10x\left[2x(x^2-4) + 2x(x^2-9)\right]}{(x^2-9)^2(x^2-4)^2} \;=\; \frac{-10(3x^4 - 13x^2 - 36)}{(x^2-9)^2(x^2-4)^2}$$

$$f''(x) \;>\; 0 \qquad 3x^4 - 13x^2 - 36 \;<\; 0 \;\Rightarrow\; -\sqrt{\frac{13+\sqrt{601}}{6}} \;<\; x \;<+\sqrt{\frac{13+\sqrt{601}}{6}}$$



E il grafico :