

Studiare le seguenti funzioni:

$$1. f(x) = \frac{x+2}{x^2+x+2}$$

$$2. f(x) = \frac{x^2-7x+6}{x^2-5x+6}$$

$$3. f(x) = \frac{\sqrt{x^2-5x-6}}{2x}$$

$$4. f(x) = \sqrt{\frac{x+1}{(x-1)(x+2)}}$$

$$5. f(x) = \frac{x-1}{\sqrt{x^2-1}}$$

$$6. f(x) = \frac{1}{(\sqrt{x-1})^4 \sqrt{x^3}}$$

$$7. f(x) = \frac{\sqrt{x^2+1}}{\sqrt{x}}$$

$$8. f(x) = \sqrt{\frac{x+1}{4-x^2}}$$

$$9. f(x) = \sqrt[3]{(x-2)^2}$$

$$10. f(x) = x - \sqrt{x^2-1}$$

$$11. f(x) = e^{\sqrt{x^2-1}}$$

$$12. f(x) = \log(x^2 - 3x - 4)$$

$$13. f(x) = \log\sqrt{e^x - 10}$$

$$14. f(x) = \sqrt{\frac{\frac{x+1}{e^{x-4}}}{\log(x^2-1)}}$$

$$15. f(x) = \log\sqrt{\frac{x^2}{1+x}}$$

$$16. f(x) = \frac{e^x - e^{-x}}{2}$$

$$17. f(x) = \frac{e^{-x}}{x^2+1}$$

$$18. f(x) = \frac{x-1}{\log(x-1)-1}$$

$$19. f(x) = xe^{\frac{1}{1+\log x}}$$

$$20. f(x) = \log\frac{\sqrt{x^2+1}-1}{x}$$

$$21. f(x) = \frac{x^2}{|x^2-1|}$$

$$22. f(x) = \sqrt{1 + |x^2 - 1|}$$

$$23. f(x) = \frac{e^{-x}}{|x-1|}$$

$$24. f(x) = \sqrt{2 + \log|x|}$$

$$25. f(x) = 1 - e^{-\left|\frac{x}{1-x}\right|}$$

$$26. f(x) = \left|\frac{x+1}{x-2}\right|$$

$$27. f(x) = \frac{1}{x^2+|x-2|}$$

$$28. f(x) = |x-1| + |x-2|$$

$$29. f(x) = \frac{|x+4|+x}{4-x^2}$$

$$30. f(x) = \log\left(\frac{1}{2} + \frac{1}{|x^2-x|}\right)$$

Soluzioni

1. $[D = \mathbb{R}; f(x) > 0 \Rightarrow x > -2; A(0; 1) B(-2; 0); \text{Asintoti: } y = 0]$
2. $\left[D = \mathbb{R} - \{2, 3\}; f(x) > 0 \Rightarrow x < 1 \vee 2 < x < 3 \vee x > 6; \begin{matrix} A(0; 1) & y = 1 \\ B(1; 0) & \\ C(6; 0) & x = 3 \end{matrix}; \text{Asintoti: } \right]$
3. $\left[D =]-\infty; -1[\cup [6; +\infty[; f(x) > 0 \Rightarrow x > 6; \begin{matrix} A(-1; 0) \\ B(6; 0) \end{matrix}; \text{Asintoti: } \begin{matrix} y = -\frac{1}{2} \\ y = \frac{1}{2} \end{matrix} \right]$
4. $\left[D =]-2; -1[\cup]1; +\infty[; f(x) > 0 \Rightarrow \forall x \in D - \{-1\}; A(-1; 0); \text{Asintoti: } \begin{matrix} y = 0 \\ x = -2 \\ x = 1 \end{matrix} \right]$
5. $[D =]-\infty; -1[\cup]1; +\infty[; f(x) > 0 \Rightarrow x > 1; A(1; 0); \text{Asintoti: } y = -1 \quad y = 1 \quad x = -1]$
6. $[D =]0; 1[\cup]1; +\infty[; f(x) > 0 \Rightarrow x > 1; \text{Asintoti: } x = 0 \quad x = 1 \quad y = 0]$
7. $[D = \mathbb{R}^+; f(x) > 0 \Rightarrow \forall x \in D; \text{Asintoti: } x = 0]$
8. $[D =]-\infty; -2[\cup [-1; 2[; f(x) > 0 \Rightarrow \forall x \in D - 1]; A(-1; 0) B\left(0; \frac{1}{2}\right); \text{Asintoti: } \left\| \begin{matrix} x = -2 \\ x = 2 \end{matrix} \right\|]$
9. $[D = \mathbb{R}; f(x) > 0 \Rightarrow \forall x \in D; A(0; \sqrt[3]{4}) B(2; 0)]$
10. $\left[D =]-\infty; -1[\cup [1; +\infty[; \begin{matrix} f(x) > 0 \Rightarrow x > 1 \\ f(x) < 0 \Rightarrow x < -1 \end{matrix}; \text{Asintoti: } y = 0 \right]$
11. $[D =]-\infty; -1[\cup [1; +\infty[; f(x) > 0 \Rightarrow \forall x \in D]$
12. $\left[D =]-\infty; -1[\cup]4; +\infty[; f(x) > 0 \Rightarrow x < \frac{3-\sqrt{29}}{2} \vee x > \frac{3+\sqrt{29}}{2}; \text{Asintoti: } x = -1 \quad x = 4 \right]$
13. $[D =]\log 10; +\infty[; f(x) > 0 \Rightarrow x > \log 11; A(\log 11; 0); \text{Asintoti: } x = \log 10]$
14. $[D =]-\infty; -\sqrt{2}[\cup]\sqrt{2}; 4[\cup]4; +\infty[; f(x) > 0 \Rightarrow \forall x \in D; \text{Asintoti: } y = 0 \quad x = 4(Dx)]$
15. $\left[D =]-1; 0[\cup]0; +\infty[; f(x) > 0 \Rightarrow -1 < x < \frac{1-\sqrt{5}}{2} \vee x > \frac{1+\sqrt{5}}{2}; \left\| \begin{matrix} A\left(\frac{1-\sqrt{5}}{2}; 0\right) \\ B\left(\frac{1+\sqrt{5}}{2}; 0\right) \end{matrix} \right\|; \text{Asintoti: } \left\| \begin{matrix} x = -1 \\ x = 0 \end{matrix} \right\| \right]$
16. $[D =]-\infty; +\infty[; f(x) > 0 \Rightarrow x > 0; A(0; 0)]$
17. $[D = \mathbb{R}; f(x) > 0 \Rightarrow \forall x \in D > 1; A(0; 1); \text{Asintoti: } y = 0 (Dx)]$
18. $\left[D =]1; e + 1[\cup]e + 1; +\infty[; f(x) > 0 \Rightarrow x > e + 1; \text{Asintoti: } x = e + 1 \left(\begin{matrix} \text{tralasciare} \\ \text{asintoti} \\ \text{orizzontali} \end{matrix} \right) \right]$
19. $[D =]0; \frac{1}{e}[\cup]\frac{1}{e}; +\infty[; f(x) > 0 \Rightarrow \forall x \in D; \text{Asintoti: } x = \frac{1}{e}]$
20. $\left[D =]0; +\infty[; \left\| \begin{matrix} f(x) \geq 0 \Rightarrow \text{impossibile} \\ f(x) < 0 \Rightarrow \forall x \in D \end{matrix} \right\| \right]$
21. $\left[D = \mathbb{R} - \{\pm 1\}; f(x) > 0 \Rightarrow \forall x \in D - \{0\}; A(0; 0); \text{Asintoti: } y = 1 \quad \begin{matrix} x = 1 \\ x = -1 \end{matrix} \right]$
22. $[D = \mathbb{R}; f(x) > 0 \Rightarrow \forall x \in D; A(0; \sqrt{2})]$
23. $[D = \mathbb{R} - \{1\}; f(x) > 0 \Rightarrow \forall x \in D; A(0; 1); \text{Asintoti: } y = 0 (Dx) \quad x = -1]$
24. $[D =]-\infty; -e^{-2}[\cup]e^{-2}; +\infty[; f(x) > 0 \Rightarrow \forall x \in D; A(-e^{-2}; 0) B(e^{-2}; 0)]$
25. $\left[D = \mathbb{R} - \{1\}; \begin{matrix} f(x) = 0 \Rightarrow x = 0 \\ f(x) > 0 \Rightarrow \forall x \in D - \{0\} \end{matrix}; A(0; 0); \text{Asintoti: } y = 1 - \frac{1}{e} \right]$
26. $\left[D = ; \begin{matrix} f(x) = 0 \Rightarrow x = -1 \\ f(x) > 0 \Rightarrow \forall x \in D - \{-1\} \end{matrix}; A\left(0; \frac{1}{2}\right) B(-1; 0); \text{Asintoti: } y = 1 \quad x = 2 \right]$
27. $[D = \mathbb{R}; f(x) > 0 \Rightarrow \forall x \in D; A\left(0; \frac{1}{2}\right); \text{Asintoti: } y = 0]$
28. $[D = \mathbb{R}; f(x) > 0 \Rightarrow \forall x \in D; A(0; 3)]$
29. $[D = \mathbb{R} - \{\pm 2\}; f(x) > 0 \Rightarrow x < 2 \quad x \neq -2; A(0; 1); \text{Asintoti: } y = 0 \quad x = 2]$
30. $\left[D = \mathbb{R} - \{0; 1\}; f(x) > 0 \Rightarrow -1 < x < 2; A(-1; 0) B(2; 0); \text{Asintoti: } y = \log \frac{1}{2} \quad \begin{matrix} x = 0 \\ x = -1 \end{matrix} \right]$