1. $\dfrac{sen2x}{1-\cos 2x}=\cot g^{2}x$

2. $\dfrac{\cos x}{1+senx}+cotg\,x=2\cos x$

3. $\dfrac{4sen^{2}x\cos^{2}x}{sen2x}=-\sqrt{3}senx$

4. $cotg\,(45°+x)=\cos x-senx$

5. $2\cos x\,sen2x\;tgx-\cos x\,tgx=2\cos x\,sen2x-\cos x$

6. $\cos x=\dfrac{sen^{2}x-\sqrt{3}sen2x}{\cos x}$

7. $tg2x-3tgx=0$

8. $\dfrac{\cos(45°-x)-\cos(45°+x)}{sen2x}=\dfrac{\sqrt{6}}{3}$

9. $cotg\,x-tgx=\dfrac{2\cos 2x}{1-\cos 2x}$

10. $tg(x+60°)+tgx=0$

11. $\dfrac{senx+1}{senx}+cotg^{2}\,x=2$

12. $cotg^{2}\,x=\dfrac{2+3\cos x}{1-\cos x}$

1) $\dfrac{\operatorname{sen} 2x}{1-\cos 2x} = \operatorname{cotg}^2 x$ $\quad 1-\cos 2x \neq 0 \quad \cos 2x \neq 1 \quad 2x \neq 0° + k360°$

$\underline{x \neq k180°}$

$\dfrac{2\operatorname{sen}x\cos x}{1-\cos^2 x+\operatorname{sen}^2 x} = \dfrac{\cos^2 x}{\operatorname{sen}^2 x} \qquad \dfrac{\cancel{2}\,\cancel{\operatorname{sen}x}\cos x}{2\operatorname{sen}^{\cancel{2}}x} = \dfrac{\cos^2 x}{\operatorname{sen}^2 x}$

$\cos x \operatorname{sen} x = \cos^2 x \qquad \cos x \operatorname{sen} x - \cos^2 x = 0$

$\cos x\left[\operatorname{sen} x - \cos x\right] = 0 \qquad \cos x = 0 \ \vee \ \operatorname{sen} x - \cos x = 0$

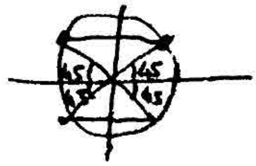
$\cos x = 0 \ \vee \ \operatorname{tg} x - 1 = 0$

$\boxed{x_1 = 90° + k180° \ \vee \ x_2 = 45° + k180°}$

2) $\dfrac{\cos x}{1+\operatorname{sen}x} + \operatorname{cotg} x = 2\cos x$ $\quad$ C.E: $1+\operatorname{sen}x \neq 0 \quad \operatorname{sen}x \neq -1 \quad \underline{x \neq 270° + k360}$

$\dfrac{\cos x}{1+\operatorname{sen}x} + \dfrac{\cos x}{\operatorname{sen}x} = 2\cos x \Rightarrow \cos x\operatorname{sen}x + \cos x + \cos x \operatorname{sen}x = 2\cos x \operatorname{sen}x\,(1+\operatorname{sen}x)$

$\cancel{2\cos x \operatorname{sen}x} + \cos x = \cancel{2\cos x\operatorname{sen}x} + 2\cos x\operatorname{sen}^2 x$



$\cos x - 2\cos x \operatorname{sen}^2 x = 0$

$\cos x\left(1 - 2\operatorname{sen}^2 x\right) = 0 \qquad \cos x = 0 \ \vee \ \operatorname{sen}^2 x = \dfrac{1}{2}$

$\cos x = 0 \ \vee \ \operatorname{sen}x = \pm\sqrt{\dfrac{1}{2}} = \pm\dfrac{1}{\sqrt{2}} = \pm\dfrac{\sqrt{2}}{2}$

$x_1 = 90° + k180° \ \vee$

$x_2 = 45° + k360°$
$x_3 = 135° + k360°$
$x_4 = 225° + k360°$
$x_5 = 315° + k360°$

OPPURE $\Rightarrow$ $x = 45° + k$; $x = 135° + k$

$\Downarrow$ OPPUR

$\boxed{x = \pm 45° + k180}$

$\Downarrow$ Essendo 270° non accettabile

$\Rightarrow \boxed{x_1 = 90° + k360°}$

3) $\dfrac{4\operatorname{sen}^2 x\cos^2 x}{\operatorname{sen}2x} = -\sqrt{3}\operatorname{sen}x$ $\quad$ C.E. $\operatorname{sen}2x \neq 0 \quad 2x \neq 0° + k180° \quad \underline{x \neq k90°}$

$\dfrac{\cancel{4}^{2}\operatorname{sen}^{\cancel{2}}x\cos^{\cancel{2}}x}{\cancel{2\operatorname{sen}x\cos x}} = -\sqrt{3}\operatorname{sen}x \Rightarrow 2\operatorname{sen}x\cos x + \sqrt{3}\operatorname{sen}x = 0 \qquad \operatorname{sen}x\left(2\cos x + \sqrt{3}\right) = 0$

$\operatorname{sen}x = 0 \ \vee \ \cos x = -\dfrac{\sqrt{3}}{2}$

$\Downarrow$

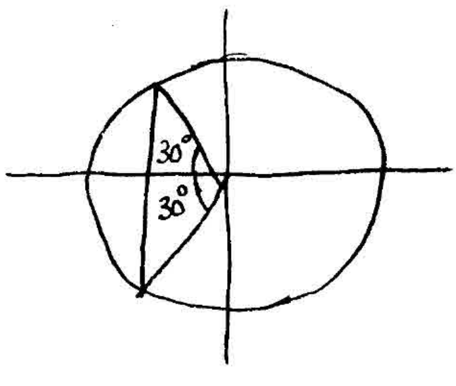
$x_1 = 0° + k180°$

$\Downarrow$

NON ACCETTAB

$\Downarrow$

$\boxed{\begin{array}{l} x_2 = 150° + k360° \\ x_3 = 210° + k360° \end{array}}$



4) $\cot g(45°+x) = \cos x - \operatorname{sen} x$ C.E. $45° + x \neq 0° + K180° \Rightarrow$

$\underline{x \neq -45° + K180°}$

$$\frac{\cot g 45° \cot g x - 1}{\cot g x + \cot g 45°} = \cos x - \operatorname{sen} x$$

$$\frac{\cot g x - 1}{\cot g x + 1} = \cos x - \operatorname{sen} x \qquad \frac{\frac{\cos x}{\operatorname{sen} x} - 1}{\frac{\cos x}{\operatorname{sen} x} + 1} = \cos x - \operatorname{sen} x$$

$$\frac{\frac{\cos x - \operatorname{sen} x}{\operatorname{sen} x}}{\frac{\cos x + \operatorname{sen} x}{\operatorname{sen} x}} = \cos x - \operatorname{sen} x \qquad \frac{\cos x - \operatorname{sen} x}{\cancel{\operatorname{sen} x}} \cdot \frac{\cancel{\operatorname{sen} x}}{\cos x + \operatorname{sen} x} = \cos x - \operatorname{sen} x$$

$$\frac{\cos x - \operatorname{sen} x}{\cos x + \operatorname{sen} x} = \cos x - \operatorname{sen} x \qquad \cos x - \operatorname{sen} x = (\cos x - \operatorname{sen} x)(\cos x + \operatorname{sen} x)$$

$$(\cos x - \operatorname{sen} x) - (\cos x - \operatorname{sen} x)(\cos x + \operatorname{sen} x) = 0$$

$$(\cos x - \operatorname{sen} x)\left[1 - \cos x - \operatorname{sen} x\right] = 0$$

$\underbrace{\cos x - \operatorname{sen} x = 0}_{\text{lineare omogenea}} \quad \vee \quad \underbrace{1 - \cos x - \operatorname{sen} x = 0}_{\text{lineare completa}} \Rightarrow 1 - \operatorname{tg} x = 0 \ \vee \ \operatorname{sen} x + \cos x - 1 = 0$

$\Downarrow$

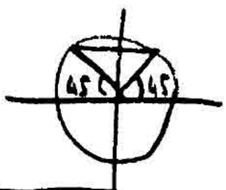
$\sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

$\operatorname{tg} x = 1 \ \vee \ \frac{1}{\sqrt{2}} \operatorname{sen} x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2} \operatorname{sen} x + \frac{\sqrt{2}}{2} \cos x = \frac{\sqrt{2}}{2}$

$\boxed{x_1 = 45° + K180°}$

$\vee \ \cos 45° \operatorname{sen} x + \operatorname{sen} 45° \cos x = \frac{\sqrt{2}}{2}$

$$\operatorname{sen}(x + 45°) = \frac{\sqrt{2}}{2}$$



$x_2 + \cancel{45°} = \cancel{45°} + K360°$ $\qquad \boxed{x_2 = K360°}$

$x_3 + 45° = 135° + K360°$ $\qquad \boxed{x_3 = 90° + K360°}$

5) $2\cos x \,\text{sen}\, 2x \,\text{tg}\, x - \cos x \,\text{tg}\, x = 2\cos x \,\text{sen}\, 2x - \cos x$ C.E: $x \neq 90° + K \cdot 180°$

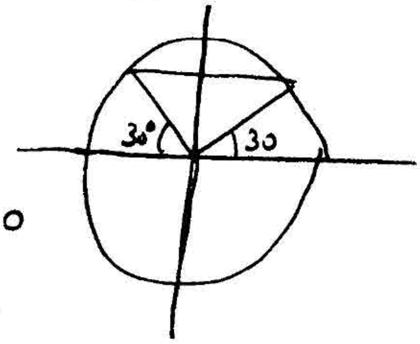
$\cos x \,\text{tg}\, x \left[2\,\text{sen}\, 2x - 1\right] = \cos x \left[2\,\text{sen}\, 2x - 1\right]$

$(2\,\text{sen}\, 2x - 1)(\cos x \,\text{tg}\, x - \cos x) = 0$

$(2\,\text{sen}\, 2x - 1)\cos x\,(\text{tg}\, x - 1) = 0$

$2\,\text{sen}\, 2x - 1 = 0 \;\vee\; \cos x = 0 \;\vee\; \text{tg}\, x - 1 = 0$

$\text{sen}\, 2x = \frac{1}{2} \;\vee\; \cos x = 0 \;\vee\; \text{tg}\, x = 1$



$2x_1 = 30° + K360°$
$2x_2 = 150° + K360°$

$\Downarrow$
$x = 90° + K180°$ NON ACCETTABILE

$x_3 = 45° + K180°$

$$\Rightarrow \boxed{\begin{array}{ll} x_1 = 15° + K180° & x_3 = 45° + K180° \\ x_2 = 75° + K180° & \end{array}}$$

6) $\cos x = \dfrac{\text{sen}^2 x - \sqrt{3}\,\text{sen}\, 2x}{\cos x}$ C.E $\cos x \neq 0$ $x \neq 90° + K180°$

$\cos^2 x = \text{sen}^2 x - \sqrt{3}\,\text{sen}\, 2x \;\Rightarrow\; \cos^2 x = \text{sen}^2 x - \sqrt{3} \cdot 2\,\text{sen}\, x \cos x$

$\Rightarrow \underbrace{\cos^2 x - \text{sen}^2 x + 2\sqrt{3}\,\text{sen}\, x \cos x = 0}_{\text{omogenea}} \;\Rightarrow\; 1 - \text{tg}^2 x + 2\sqrt{3}\,\text{tg}\, x = 0$

$\text{tg}^2 x - 2\sqrt{3}\,\text{tg}\, x - 1 = 0 \;\Rightarrow\; \Delta = 12 + 4 = 16$

$\text{tg}\, x = \dfrac{2\sqrt{3} \pm 4}{2} = \begin{cases} \dfrac{2\sqrt{3}+4}{2} = \sqrt{3}+2 & x_1 = \text{arctg}(\sqrt{3}+2) + K180° \\[2ex] \dfrac{2\sqrt{3}-4}{2} = \sqrt{3}-2 & x_2 = \text{arctg}(\sqrt{3}-2) + K180° \end{cases}$

$$\boxed{x_1 = 75° + K180° \;\vee\; x_2 = -15° + K180°}$$

7) $\text{tg}\, 2x - 3\,\text{tg}\, x = 0$ C.E $x \neq 90° + K180°$

$\dfrac{2\,\text{tg}\, x}{1 - \text{tg}^2 x} - 3\,\text{tg}\, x = 0$ $2\,\text{tg}\, x - 3\,\text{tg}\, x(1 - \text{tg}^2 x) = 0$

$2\,\text{tg}\, x - 3\,\text{tg}\, x + 3\,\text{tg}^3 x = 0 \;\Rightarrow\; -\text{tg}\, x + 3\,\text{tg}^3 x = 0 \;\Rightarrow\; 3\,\text{tg}^3 x - \text{tg}\, x = 0$

$\text{tg}\, x(3\,\text{tg}^2 x - 1) = 0 \;\Rightarrow\; \text{tg}\, x = 0 \;\vee\; \text{tg}^2 x = \frac{1}{3} \;\Rightarrow\; \text{tg}\, x = 0 \;\vee\; \text{tg}\, x = \pm\frac{\sqrt{3}}{3}$

$$\boxed{x_1 = 0° + K180° \;\vee\; x = \pm 30° + K180°}$$

8) $\dfrac{\cos(45^\circ - x) - \cos(45^\circ + x)}{\operatorname{sen} 2x} = \dfrac{\sqrt{6}}{3}$ C.E : $\operatorname{sen} 2x \neq 0$ $2x \neq 0^\circ + K180^\circ$

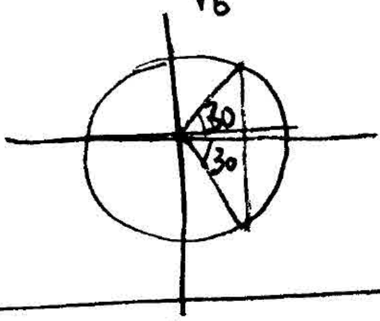
$\underline{x \neq K90^\circ}$

$$\frac{[\cos 45^\circ \cos x + \operatorname{sen} 45^\circ \operatorname{sen} x] - [\cos 45^\circ \cos x - \operatorname{sen} 45^\circ \operatorname{sen} x]}{2 \operatorname{sen} x \cos x} = \frac{\sqrt{6}}{3}$$

$$\frac{\cancel{\frac{\sqrt{2}}{2}\cos x} + \frac{\sqrt{2}}{2}\operatorname{sen} x - \cancel{\frac{\sqrt{2}}{2}\cos x} + \frac{\sqrt{2}}{2}\operatorname{sen} x}{2 \operatorname{sen} x \cos x} = \frac{\sqrt{6}}{3}$$

$$\frac{\cancel{2}\frac{\sqrt{2}}{2}\cancel{\operatorname{sen} x}}{\cancel{2}\cancel{\operatorname{sen} x}\cos x} = \frac{\sqrt{6}}{3} \Rightarrow \frac{\frac{\sqrt{2}}{2}}{\cos x} = \frac{\sqrt{6}}{3} \Rightarrow 3\frac{\sqrt{2}}{2} = \sqrt{6}\cos x$$

$$\Rightarrow \cos x = \frac{\frac{3\sqrt{2}}{2}}{\sqrt{6}} = \frac{3\sqrt{2}}{2} \cdot \frac{1}{\sqrt{6}} = \frac{3\sqrt{2}}{2\sqrt{6}} = \frac{3\sqrt{2}\cdot\sqrt{6}}{2\sqrt{6}\sqrt{6}} = \frac{3\sqrt{12}}{12} = \frac{\cancel{6}\sqrt{3}}{\cancel{12}_{2}} = \frac{\sqrt{3}}{2}$$



$\boxed{\begin{array}{l} x_1 = 30^\circ + K360^\circ \\ x_2 = 300 + K360^\circ \end{array}} \Rightarrow$ oppure $\Rightarrow \boxed{x = \pm 30^\circ + K360^\circ}$

---

9) $\cot g\, x - tg\, x = \dfrac{2\cos 2x}{1 - \cos 2x}$ C.E $\underline{x \neq 0^\circ + K180^\circ}$

$\underline{x \neq 90^\circ + K180^\circ}$

$1 - \cos 2x \neq 0$ $\cos 2x \neq 1$ $2x \neq 0^\circ + K360^\circ$

$x \neq K180^\circ$

$$\frac{\cos x}{\operatorname{sen} x} - \frac{\operatorname{sen} x}{\cos x} = \frac{2[\cos^2 x - \operatorname{sen}^2 x]}{1 - [\cos^2 x - \operatorname{sen}^2 x]}$$

$$\frac{\cos^2 x - \operatorname{sen}^2 x}{\operatorname{sen} x \cos x} = \frac{2\cos^2 x - 2\operatorname{sen}^2 x}{1 - \cos^2 x + \operatorname{sen}^2 x}$$

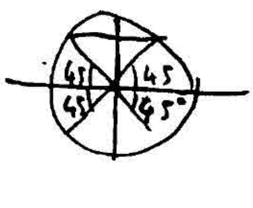
$$\frac{1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x}{\operatorname{sen} x \cos x} = \frac{2[1 - \operatorname{sen}^2 x] - 2\operatorname{sen}^2 x}{\operatorname{sen}^2 x + \operatorname{sen}^2 x} \Rightarrow \frac{1 - 2\operatorname{sen}^2 x}{\operatorname{sen} x \cos x} = \frac{2 - 2\operatorname{sen}^2 x - 2\operatorname{sen}^2 x}{2\operatorname{sen}^2 x}$$

$$\frac{1 - 2\operatorname{sen}^2 x}{\operatorname{sen} x \cos x} = \frac{\cancel{2} - \cancel{4}^{2}\operatorname{sen}^2 x}{\cancel{2}\operatorname{sen}^2 x} \Rightarrow \frac{1 - 2\operatorname{sen}^2 x}{\operatorname{sen} x \cos x} = \frac{1 - 2\operatorname{sen}^2 x}{\operatorname{sen}^2 x}$$

$$\operatorname{sen} x(1 - 2\operatorname{sen}^2 x) - \cos x(1 - 2\operatorname{sen}^2 x) = 0 \Rightarrow (1 - 2\operatorname{sen}^2 x)(\operatorname{sen} x - \cos x) = 0$$

$$\Rightarrow 1 - 2\operatorname{sen}^2 x = 0 \quad \vee \quad \operatorname{sen} x - \cos x = 0 \quad \Rightarrow \operatorname{sen}^2 x = \frac{1}{2} \quad \vee \quad tg\, x - 1 = 0$$

$\operatorname{sen} x = \pm\sqrt{\dfrac{1}{2}} = \pm\dfrac{\sqrt{2}}{2} \Rightarrow$



$\boxed{x_1 = \pm 45^\circ + K180^\circ}$

$tg\, x = 1$

$x_2 = 45^\circ + K180^\circ$

10) $tg(x+60°) + tgx = 0$ C.E $x+60° \neq 90° + K180°$

$\underline{x \neq 30° + K180°}$

$\underline{x \neq 90° + K180°}$

$$\frac{tgx + tg60°}{1 - tgx\, tg60} + tgx = 0$$

$$\frac{tgx + \sqrt{3}}{1 - tgx\sqrt{3}} + tgx = 0$$

$$\cancel{tgx} + \sqrt{3} + \cancel{tgx} - tg^2x \cdot \sqrt{3} = 0$$

$$\sqrt{3}\,tg^2x - 2\,tgx - \sqrt{3} = 0 \qquad \Delta = 4 + 12 = 16$$

$$tgx = \frac{2 \pm 4}{2\sqrt{3}} = \begin{cases} \dfrac{6}{2\sqrt{3}} = \dfrac{3}{\sqrt{3}} = \dfrac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3} \\ \dfrac{-\cancel{2}}{\cancel{2}\sqrt{3}} = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3} \end{cases}$$

$$\boxed{\begin{array}{l} x_1 = 60° + K180° \\ x_2 = -30° + K180° \end{array}}$$

11) $\dfrac{senx + 1}{senx} + cotg^2x = 2$ C.E $senx \neq 0$ $\underline{x \neq 0° + K180°}$

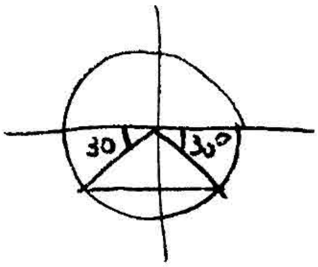
$x \neq 0° + K180°$

$$\frac{senx+1}{senx} + \frac{cos^2x}{sen^2x} = 2 \implies sen^2x + senx + cos^2x = 2\,sen^2x$$

$$\cancel{sen^2x} + senx + 1 - \cancel{sen^2x} - 2\,sen^2x = 0$$

$$2\,sen^2x - senx - 1 = 0 \qquad \Delta = 1 + 8 = 9$$

$$senx = \frac{1 \pm 3}{4} = \begin{cases} 1 \implies \\ -\dfrac{1}{2} \implies \end{cases} \qquad \boxed{\begin{array}{l} x_1 = 90° + K360° \\ x_2 = 210° + K360° \\ x_3 = 330° + K360° \end{array}}$$



12) $cotg^2x = \dfrac{2 + 3\cos x}{1 - \cos x}$ C.E $\underline{x \neq 0° + K180°}$

$\cos x \neq 1$ $x \neq 0° + K360°$

$$\frac{cos^2x}{sen^2x} = \frac{2+3\cos x}{1-\cos x} \implies \frac{cos^2x}{1-cos^2x} = \frac{2+3\cos x}{1-\cos x} \implies \frac{cos^2x}{(1-\cos x)(1+\cos x)} = \frac{2+3\cos x}{1-\cos x}$$

$$cos^2x = (2+3\cos x)(1+\cos x) \implies \underline{cos^2x} = 2 + \underline{2\cos x} + \underline{3\cos x} + \underline{3cos^2x}$$

$$2cos^2x + 5\cos x + 2 = 0 \qquad \Delta = 25 - 16 = 9$$

$$\cos x = \frac{-5 \pm 3}{4} = \begin{cases} -2 & \underline{\text{IMPOSS.}} \\ -\dfrac{1}{2} \end{cases} \qquad \boxed{\begin{array}{l} x_1 = 120° + K360° \\ x_2 = 240° + K360° \end{array}}$$