Studiare le seguenti funzioni:

1. $f(x) = \frac{x-6}{x^2-2x+1}$

2. $f(x) = \frac{3x^2+x-4}{x^2+x+1}$

3. $f(x) = \frac{x^2-4}{x^2-1}$

4. $f(x) = \sqrt{\frac{x-1}{x+1}}$

5. $f(x) = e^{\sqrt{\frac{1+x}{x}}}$

6. $f(x) = \frac{2^{\sqrt{x^2-4}}+4}{\sqrt{e^x-e^{-x}}}$

7. $f(x) = \frac{x^2-2x+12}{ln(16-x^2)}$

8. $f(x) = 2^{\frac{x}{x-1}}$

9. $f(x) = \frac{\sqrt{8x-x^2}}{x^2-9}$

10. $f(x) = \frac{x}{x^2-3x-4}$

11. $f(x) = \frac{2x-x^2}{log_3(x-5)}$

12. $f(x) = log_2 \frac{x^2+1}{x^2-4}$

13. $f(x) = \frac{\sqrt{x}}{1-log_2 x}$

14. $f(x) = \frac{x}{1-2^x}$

15. $f(x) = 3^{\frac{2x}{x^2-1}}$

16. $f(x) = \frac{\sqrt{x+1}}{e^{x+1}}$

17. $f(x) = \frac{log_{\frac{1}{4}}(x^2-3x+3)}{x^2+4}$

18. $f(x) = \frac{\sqrt{x^2+1}}{(x-1)^2}$

19. $f(x) = \frac{log_{\frac{1}{2}}(-x^2+x)}{\sqrt{x^2+3}}$

20. $f(x) = \frac{|2x-1|}{log_{\frac{1}{4}}(3x^2-8x+7)}$

21. $f(x) = \frac{x^2+4x+3}{\sqrt{x^2-2x-8}}$

22. $f(x) = \frac{log_{\frac{1}{2}}(3-|5-3x|)}{2^{\sqrt{x}}}$

23. $f(x) = \frac{\sqrt{x-4}-\sqrt{2x+1}}{log_2(-2x^2+3x)}$

24. $f(x) = \frac{|x^2-2x|}{ln\sqrt{x}-1}$

25. $f(x) = \frac{log_{\frac{1}{3}}x^2}{\sqrt{4x-4}}$

26. $f(x) = \frac{ln\,(x-3)^2}{ln\,x+1}$

27. $f(x) = \frac{log|x-2|}{5^{x^2-4x}+1}$

28. $f(x) = \frac{log_{\frac{1}{3}}|3^{x-1}+9|}{|2x^2+5x+2|}$

29. $f(x) = \frac{x^2+\sqrt{2}}{log(1-|2+x|)}$

30. $f(x) = \frac{ln(|1-x|+1)}{|2x^2-3x|}$

Soluzioni

1. $\left[\begin{matrix} & f(x)>0 \Rightarrow x>6 \\ D=\mathbb{R}-\{1\}; & f(x)<0 \Rightarrow x<6;\ A(0;-6),\ B(6;0) \\ & f(x)=0 \Rightarrow x=6 \end{matrix}\right]$

2. $\left[\begin{matrix} & f(x)>0 \Rightarrow x<-\frac{4}{3} \vee x>1 \\ D=\mathbb{R}; & f(x)<0 \Rightarrow -\frac{4}{3}<x<1 \quad ;\ A\left(-\frac{4}{3};0\right),\ B(1;\ 0),\ C(0;\ -4) \\ & f(x)=0 \Rightarrow x=-\frac{4}{3} \vee x=1 \end{matrix}\right]$

3. $\left[\begin{matrix} & f(x)>0 \Rightarrow x<-2 \vee -1<x<1 \vee x>2 \\ Pari;\ D=\mathbb{R}-\{\pm 1\}; & f(x)<0 \Rightarrow -2<x<-1 \vee 1<x<2 \quad ;\ A(-2;0)\ B(2;0);\ C(0;4) \\ & f(x)=0 \Rightarrow x=\pm 2 \end{matrix}\right]$

4. $\left[\begin{matrix} & f(x)>0 \Rightarrow \forall\, x\in D-\{1\} \\ D=]-\infty;-1[\cup[1;+\infty[; & f(x)<0 \Rightarrow impossibile\ ;A(1;0) \\ & f(x)=0 \Rightarrow x=1 \\ & ; \end{matrix}\right]$

5. $\left[\begin{matrix} & f(x)>0 \Rightarrow \forall\, x\neq -1 \\ D=]-\infty;-1]\cup]0;+\infty[; & f(x)<0 \Rightarrow impossibile; Nessuna\ intersezione\ con\ gli\ assi \\ & f(x)=0 \Rightarrow x=-1 \end{matrix}\right]$

6. $\left[D=]2;+\infty[;\ \begin{matrix} f(x)>0 \Rightarrow \forall\, x\in D \\ f(x)\leq 0 \Rightarrow impossibile \end{matrix};\right]$

7. $\left[\begin{matrix} & f(x)>0 \Rightarrow -\sqrt{15}<x<\sqrt{15} \\ D=]-4;4[-\{\pm\sqrt{15}\}; & f(x)<0 \Rightarrow -4<x<-\sqrt{15} \vee \sqrt{15}<x<4;\ A\left(0;\frac{12}{ln16}\right) \\ & f(x)=0 \Rightarrow impossibile \end{matrix}\right]$

8. $\left[D=\mathbb{R}-\{1\};\ \begin{matrix} f(x)>0 \Rightarrow \forall\, x\in D \\ f(x)\leq 0 \Rightarrow impossibile \end{matrix};\ A(0;1)\right]$

9. $\left[\begin{matrix} & f(x)>0 \Rightarrow 3<x<8 \\ D=[0;8]-\{3\}; & f(x)<0 \Rightarrow 0<x<3 \quad ;O(0;0),A(8;0) \\ & f(x)=0 \Rightarrow x=0 \vee x=8 \end{matrix}\right]$

10. $\left[\begin{matrix} & f(x)>0 \Rightarrow -1<x<0 \vee x>4 \\ D=\mathbb{R}-\{-1;4\}; & f(x)<0 \Rightarrow x<-1 \vee 0<x<4;\ O(0;0) \\ & f(x)=0 \Rightarrow x=0 \end{matrix}\right]$

11. $\left[\begin{matrix} & f(x)>0 \Rightarrow 5<x<6 \\ D=]5;+\infty[-\{6\}; & f(x)<0 \Rightarrow x>6 \quad ;\ Nessuna\ intersezione\ con\ gli\ assi \\ & f(x)=0 \Rightarrow impossibile \end{matrix}\right]$

12. $\left[Pari;\ D =]-\infty;-2[\cup]2;+\infty[;\ \begin{matrix} f(x)>0 \Rightarrow \forall\, x \in D \\ f(x)\leq 0 \ \Rightarrow impossibile \end{matrix}\ ;\ Nessuna\ intersezione\ con\ gli\ assi\right]$

13. $\left[\begin{matrix} & f(x)>0 \Rightarrow 0<x<2 & \\ D =]0;+\infty[-\{2\};\ & f(x)<0 \ \Rightarrow x>2; & ;\ Nessuna\ intersezione\ con\ gli\ assi \\ & f(x)=0 \ \Rightarrow impossibile & \end{matrix}\right]$

14. $\left[D = \mathbb{R}-\{0\};\ \begin{matrix} f(x)\geq 0 \ \Rightarrow impossibile \\ f(x)<0 \Rightarrow \forall\, x \in D \end{matrix}\ ;\ Nessuna\ intersezione\ con\ gli\ assi\right]$

15. $\left[D = \mathbb{R}-\{\pm 1\};\ \begin{matrix} f(x)>0 \Rightarrow \forall\, x \in D \\ f(x)\leq 0 \ \Rightarrow impossibile \end{matrix};\ A(0;1)\ \right]$

16. $\left[\begin{matrix} & f(x)>0 \Rightarrow \forall\, x \in D-\{-1\} & \\ D = [-1;+\infty[;\ & f(x)<0 \ \Rightarrow impossibile & ;\ A(-1;0), B\left(0;\frac{1}{e}\right) \\ & f(x)=0 \ \Rightarrow\ x=-1 & \end{matrix}\right]$

17. $\left[\begin{matrix} & f(x)>0 \Rightarrow 1<x<2 & \\ D = \mathbb{R};\ & f(x)<0 \ \Rightarrow x<1 \vee x>2 & ;\ A(1;0), B(2;0), C\left(0;\frac{log_4 3}{4}\right) \\ & f(x)=0 \ \Rightarrow\ x=1 \vee x=2 & \end{matrix}\right]$

18. $\left[\begin{matrix} & f(x)>0 \Rightarrow \forall\, x \in D & \\ D = \mathbb{R}-\{1\};\ & f(x)<0 \ \Rightarrow impossibile & ;\ A(0;1) \\ & f(x)=0 \ \Rightarrow\ impossibile & \end{matrix}\right]$

19. $\left[D =]0;1[;\ \begin{matrix} f(x)>0 \Rightarrow \forall\, x \in D \\ f(x)\leq 0 \ \Rightarrow impossibile \end{matrix}\ ;\ Nessuna\ intersezione\ con\ gli\ assi\right]$

20. $\left[\begin{matrix} & f(x)>0 \Rightarrow impossibile & \\ D = \mathbb{R};\ & f(x)<0 \ \Rightarrow\ \forall x \in D-\left\{\frac{1}{2}\right\}; & A\left(\frac{1}{2};0\right), B\left(0;\frac{1}{log_{\frac{1}{4}} 7}\right) \\ & f(x)=0 \Rightarrow x=\frac{1}{2} & \end{matrix}\right]$

21. $\left[\begin{matrix} & f(x)>0 \Rightarrow x<-3 \ \vee x>4 & \\ D =]-\infty;-2[\cup]4;+\infty[;\ & f(x)<0 \Rightarrow\ -3<x<-2 & ;A(-3;0) \\ & f(x)=0 \Rightarrow x=-3 & \end{matrix}\right]$

22. $\left[\begin{matrix} & f(x)>0 \Rightarrow \frac{2}{3}<x<1 \vee \frac{7}{3}<x<\frac{8}{3} & \\ D = \left]\frac{2}{3};\frac{8}{3}\right[;\ & f(x)<0 \Rightarrow 1<x<\frac{7}{3} & ;A(1;0); B(\frac{7}{3};0) \\ & f(x)=0 \Rightarrow\ x=1 \vee x=\frac{7}{3} & \end{matrix}\right]$

23. $[\mathrm{D} = \emptyset]$

24. $\left[\begin{matrix} & f(x)>0 \Rightarrow x>e^2 & \\ \mathrm{D} =]0;e^2[\ \cup]e^2;+\infty[;\ & f(x)<0 \Rightarrow 0<x<2 \vee 2<x<e^2; & \mathrm{A}(2;0) \\ & f(x)=0 \Rightarrow impossibile & \end{matrix}\right]$

25. $\left[\mathrm{D} =]1; +\infty[; \begin{matrix} f(x) \geq 0 \Rightarrow impossibile \\ f(x) < 0 \Rightarrow \forall x \in D \end{matrix}; \ Nessuna\ intersezione\ con\ gli\ assi\right]$

26. $\left[\mathrm{D} =]0; \frac{1}{e}[\cup]\frac{1}{e}; 3[\cup]3; +\infty[; \begin{matrix} f(x) > 0 \Rightarrow \frac{1}{e} < x < 2 \vee x > 4 \\ f(x) < 0 \Rightarrow 0 < x < \frac{1}{e} \vee 2 < x < 3 \vee 3 < x < 4 \\ f(x) = 0 \Rightarrow x = 2 \vee x = 4 \end{matrix}; \ \mathrm{A}(4; 0)\right]$

27. $\left[\mathrm{D} = \mathbb{R} - \{2\}; \begin{matrix} f(x) > 0 \Rightarrow x < 1 \vee x > 3 \\ f(x) < 0 \Rightarrow \ 1 < x < 2 \vee 2 < x < 3 \\ f(x) = 0 \Rightarrow x = 1 \vee x = 3 \end{matrix}; \ \mathrm{A}(1; 0), \mathrm{B}(3; 0)\right]$

28. $\left[\mathrm{D} = \mathbb{R} - \left\{-2; -\frac{1}{2}\right\}; \begin{matrix} f(x) \geq 0 \Rightarrow impossibile \\ f(x) < 0 \Rightarrow \forall x \in D \end{matrix}; \ \mathrm{A}\left(0; \frac{log_{\frac{1}{3}}\frac{28}{3}}{2}\right)\right]$

29. $\left[\mathrm{D} =]-3; -2[\cup]-2; -1[; \begin{matrix} f(x) \geq 0 \Rightarrow impossibile \\ f(x) < 0 \Rightarrow \forall x \in D \end{matrix}; \ Nessuna\ intersezione\ con\ gli\ assi\right]$

30. $\left[\mathrm{D} = \mathbb{R} - \left\{0; \frac{3}{2}\right\}; \begin{matrix} f(x) > 0 \Rightarrow \forall x \neq 0, x \neq 1, x \neq \frac{3}{2} \\ f(x) < 0 \Rightarrow impossibile \\ f(x) = 0 \Rightarrow x = 1 \end{matrix}; \ Nessuna\ intersezione\ con\ gli\ assi\right]$